**9-1 Model fitting to data using LSM**

**PART I: Fitting the catalytic model to seroprevalence data to estimate the force of infection**

We will first analyze the data from the UK (“seroprevalence\_uk”):

Ignoring the maternal antibodies, write the formula for the proportion of age “a” who have ever been infected in terms of force of infection “”.

1. Assume the initial value for the force infection in the UK to be 0.12. Do you think the true value for the force infection in the UK was greater or smaller than that currently assumed? What is the current value for squared error?
2. What is the best-fitting value for the force of infection and the current value for squared error? Plot a graph of model predictions and observed data.
3. For which age groups does the model underestimate the proportion of individuals who are seropositive? For which age groups does it overestimate it?
4. According to the formula, what is the average age at infection in the UK assuming that the force of infection is independent of age?
5. Assuming that the average life expectancy (L) is 60 years, what is the R0 for this population according to the expression R0=L/A? What is the herd immunity threshold?

Fit the catalytic model to China data (“seroprevalence\_china”) to estimate the force of infection:

1. Determine the best-fitting force of infection and plot a graph of model predictions and observed data. Calculate the average age at infection, the R0 (assuming that the life expectancy is the same as that in the UK) and herd immunity.
2. How do the values for the force of infection, average age at infection, R0 and herd immunity threshold in China compare against those for the UK? Suggest possible reasons for these differences.
3. Modify the expression for the prevalence of previous infection at each age assuming that individuals are immune for the first 6 months of life as a result of maternal antibodies.
4. Assuming that individuals are immune for the first 6 months of life and are then susceptible, refit the model to estimate the force of infection in the UK and China. Plot a graph of model predictions and observed data.

Plot the graphs of –ln(Sa/Na) for China and the UK, where Sa is the number of susceptible at age “a” and Na is the number of population at age “a”.

1. According to the plots, is the assumption that the force of infection is independent of age in these populations justified? At what age does it look as though the force of infection changes in these populations?
2. Estimate the age-specific forces of infection using 2 age groups for the UK and China. Suggest reasons for the differences in the force of infection between China and the UK.

**PART Ⅱ: Fitting transmission model to prevalence data to estimate the transmission rate**

Set up the SEIR model of the transmission dynamics of measles in a closed population using differential equations.



We assume that individuals mix randomly and parameter and initial values are given as follows:

Population 6000 people

Pre-infectious period 8 days

Infectious period 7 days

Initial values (S,E,I,R)=(5999,0,1,0)

Fit the model to prevalence data to estimate the transmission rate (“incidence\_measles”):

1. What is the best-fitting value for the transmission rate? Plot a graph of model predictions and observed data.
2. Calculate R0 and herd immunity.